Detection and characterization of the singularities of functions

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Université Paris Est Créteil

Francqui chair inaugural lecture

Vrije Universiteit Brussel March 28th 2024

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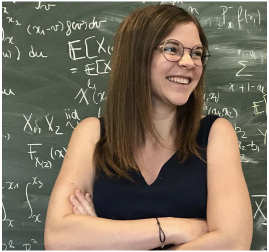

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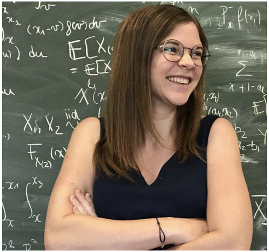
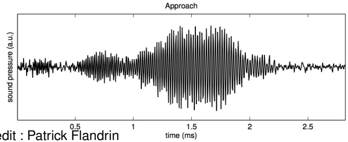


Céline Esser

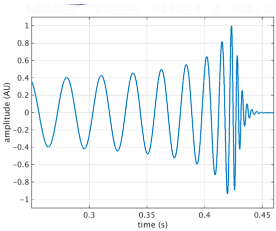
Jasson Vindas

Chirps everywhere

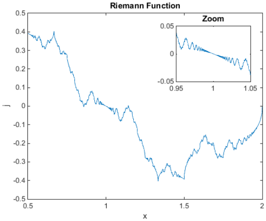


Credit : Patrick Flandrin

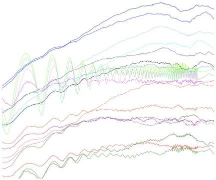
Ultrasound emitted by a bat



Gravitational wave



$$\mathcal{R}(x) = \sum_{n=1}^{\infty} \frac{\sin(\pi n^2 x)}{n^2}$$



EEG

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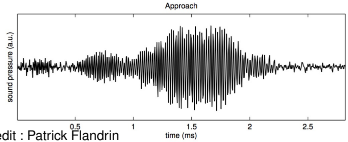

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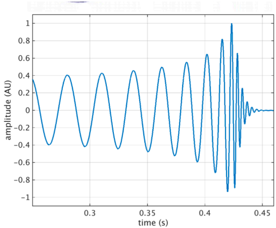

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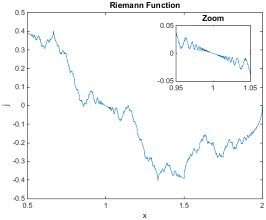

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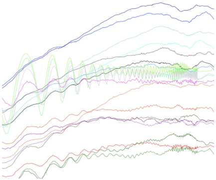


EEG

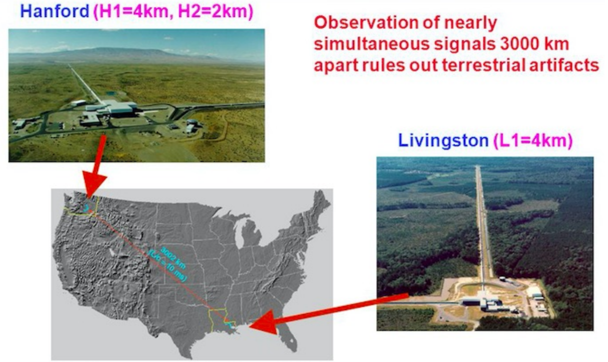
First definition : $a(t)\cos(\varphi(t))$

where $a(t)$ and $\varphi(t)$ have a *"slow and smooth"* evolution

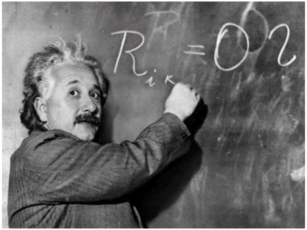
AM-FM signals (*Amplitude Modulated - Frequency Modulated*)

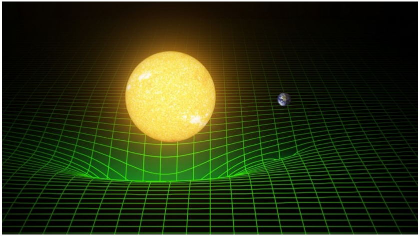
September 14th 2015 09 :50 :45 UTC

The LIGO (Laser Interferometer Gravitational-Wave Observatory) observatories in Hanford (state of Washington) and Livingston (Louisiana) performed the first detection of a gravitational wave



One minute of general relativity





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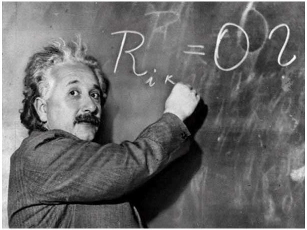

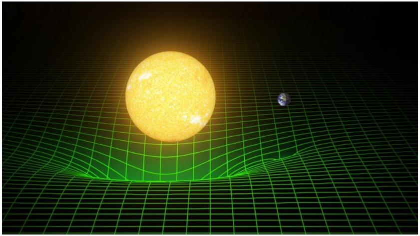

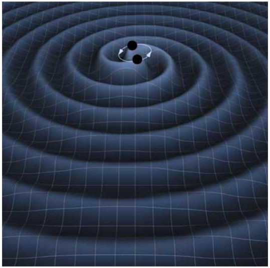

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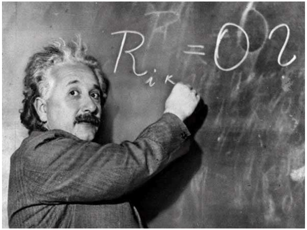
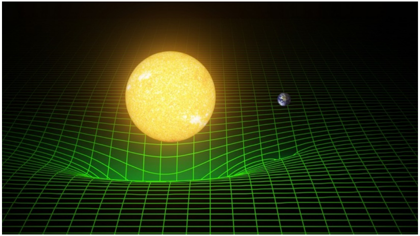
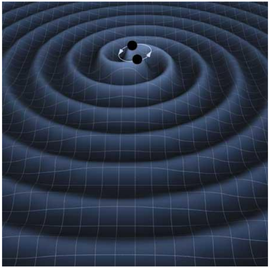
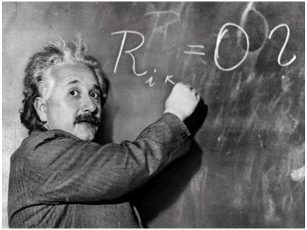


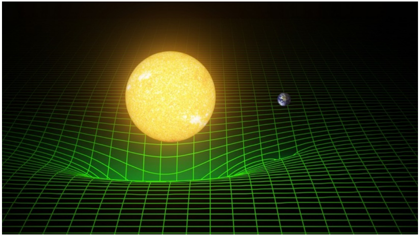


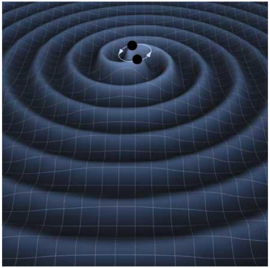


Einstein predicted the existence of gravitational waves (1916)

One minute of general relativity







Einstein predicted the existence of gravitational waves (1916)

Consequences of the detection :

- Confirmation of general relativity in extreme conditions of mass and energy

- A new astronomy

Credit : http ://www.actusf.com/spip/L-actu-des-sciences-Avril-2014.html

A few orders of magnitude

Signal emitted 1,4 billion years ago

Coalescence of 2 black holes of 36 and 29 solar masses

Energy dissipated in 0.2 seconds : 3 solar masses

Credit : http ://www.gravity.phys.uwm.edu/research/highlights/index.html ?artfile=160211-51.xml

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Size of the recorded signal before denoising : $\sim 10^{-18}$ m
Size of the gravitational wave which crossed the earth : 10^{-21} m

Radius of the hydrogen atom : 10^{-11} m
Radius of the atomic nucleus : 10^{-15} m

Scientific challenges

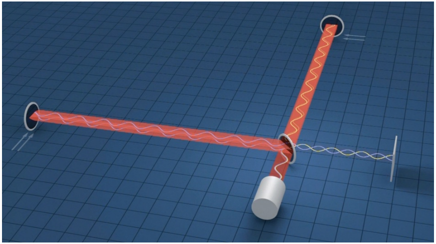
Instrumental and Physics

An extreme sophistication of the experimental device

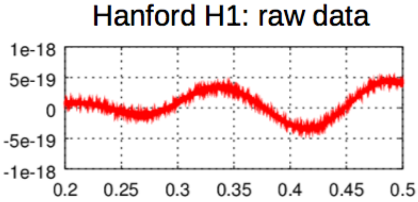
Michelson Interferometer with 2 arms of 4 km length

The laser beam is reflected several hundreds of times

Rainer Weiss



The denoising algorithm : Frequency filtering



Gravitational wave GW150914 recorded by :

LIGO Hanford (H1, left) and Livingston (L1, right) detectors

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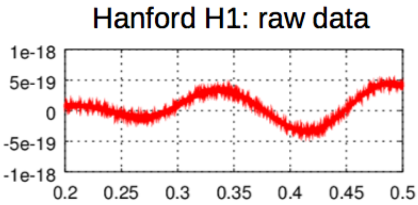

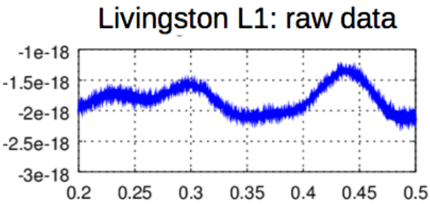

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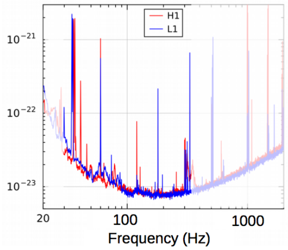
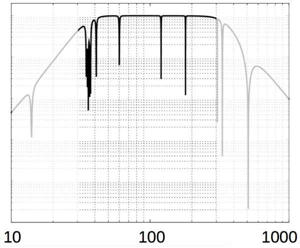
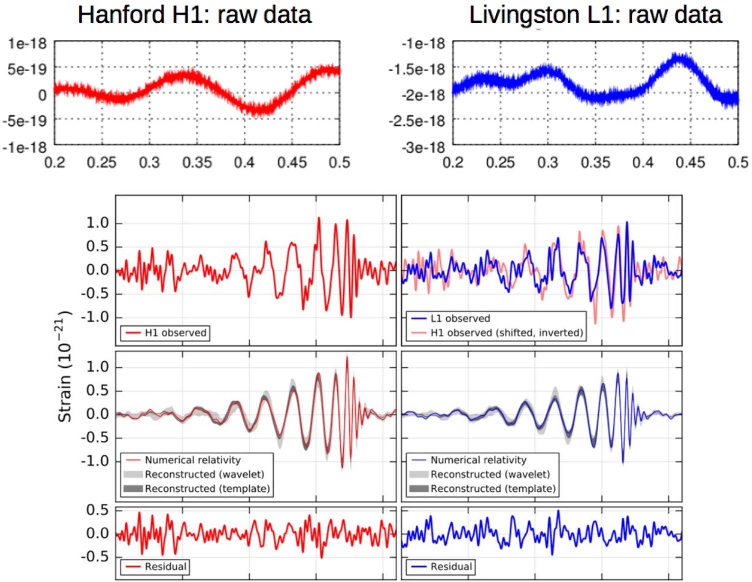


Frequencies in the data

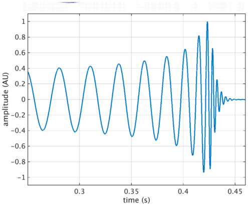


Fourier transform of the filter

The result of Fourier filtering



The Short-Time Fourier Transform (STFT)



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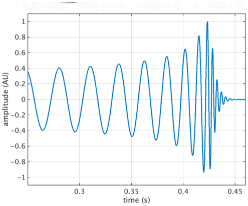

Let φ be a Gaussian function

The short-time Fourier transform of f is

$$G_f(x,\xi) = \int_{\mathbb{R}} f(x)\varphi(t-x)\, e^{-2i\pi t\xi} dt$$

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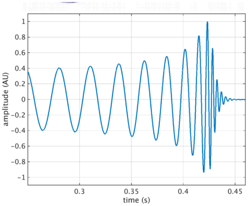

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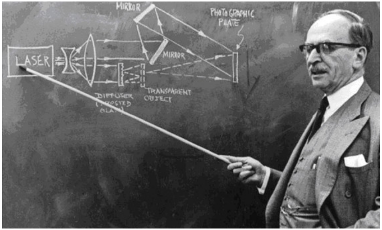
$$f(t) = \int \int G_f(x,\xi)\, e^{2i\pi\xi t}\, \varphi(t-x)\, d\xi\, dx$$



D. Gabor

1971 Nobel prize laureate for holography

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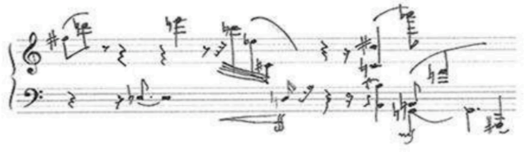

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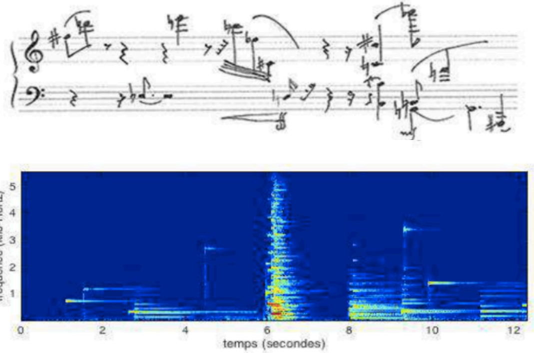
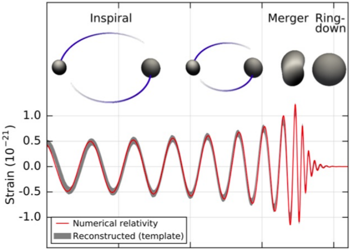
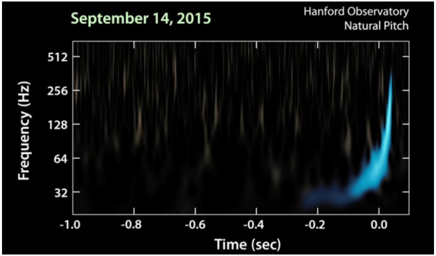


Credit : Patrick Flandrin

STFT of gravitational waves





B. Torresani

P. Flandrin

E. Chassande-Mottin

Time-frequency orthonormal bases ?

(1) $\quad 1_{[k,k+1[}(x)\, e^{2i\pi n x} \qquad k,\ n \in \mathbb{Z}$

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Gabor's dream of "logons" : Expand any signal on the

$$\varphi(x-k)\; e^{2i\pi nx} \qquad k, n \in \mathbb{Z}$$

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The Balian-Low theorem (1981) : If

$$\int (1+t^2)|g(t)|^2 dt < \infty \quad \text{et} \quad \int (1+\xi^2)|\hat{g}(\xi)|^2 d\xi < \infty$$

then any system of the form

$$g(x - a\,k)\, e^{i\,b\,n\,x} \qquad k,\ n \in \mathbb{Z}$$

is either incomplete or over-complete, and never is a basis of $L^2(\mathbb{R})$

(1) is compatible with the Balian-Low theorem

Beyond the Balian-Low theorem

T. Steger : A Riesz basis $\varphi_n(x)$ of $L^2(\mathbb{R})$ cannot simultaneously verify

$$\exists a_n,\, b_n : \int (1+|t-a_n|^2)|\varphi_n(t)|^2 dt < \infty \text{ and } \int (1+|\xi-b_n|^2)|\widehat{\varphi_n}(\xi)|^2 d\xi < \infty$$

(“strong” uncertainty principle for bases)

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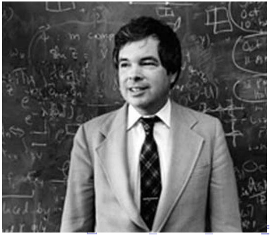


In 1987 K. Wilson (1982 Nobel laureate for renormalization theory) proposed a way :

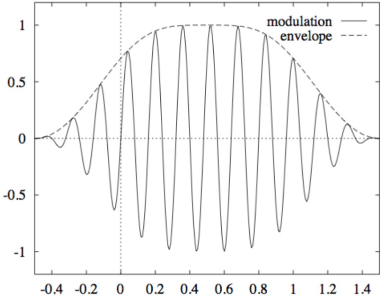
Allow a double Fourier localization around two frequencies of same amplitude and opposite signs

Wilson bases

Wilson bases (I. Daubechies, S. J., J.-L. Journé, 1991) are orthonormal bases of the form

$$\varphi_{0,n}(t) = \varphi(t-n) \qquad n \in \mathbb{Z},$$

$$\varphi_{l,n}(t) = \begin{cases} \sqrt{2}\varphi\left(t - \frac{n}{2}\right)\cos(2\pi l t) & \text{if } l+n \in 2\mathbb{Z}, \\ \sqrt{2}\varphi\left(t - \frac{n}{2}\right)\sin(2\pi l t) & \text{if } l+n \in 2\mathbb{Z}+1 \end{cases}$$



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- φ and $\widehat{\varphi}$ can both have exponential decay
- $\widehat{\varphi}$ can be compactly supported

J.-L. Journé († April 2016)

I. Daubechies

Coherent Wave Burst

Algorithm due to S. Klimenko and his collaborators based on a union of dilated Wilson bases

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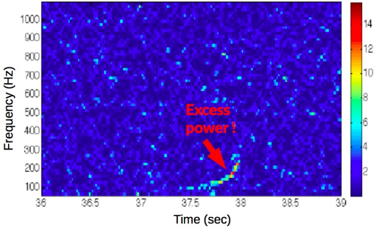
Algorithm due to S. Klimenko and his collaborators based on a union of dilated Wilson bases

Discrete time-frequency analysis of the gravitational wave



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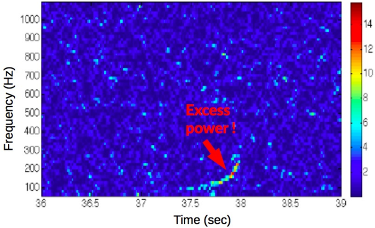


Advantages :

- Sparse representation of gravitational waves
- Fast decomposition algorithms (based on FFT)

Bottlenecks :

- Stability properties of the algorithm
- Which functions are sparse on such redundant bases ?

EEG

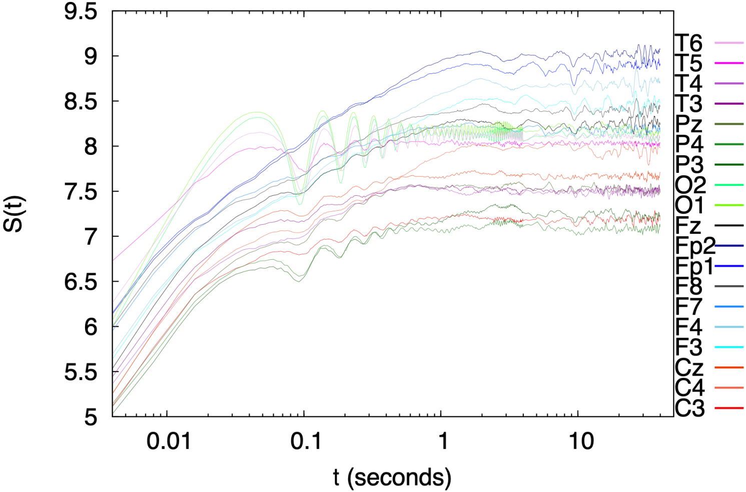
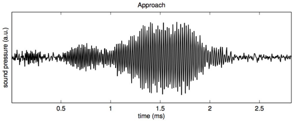


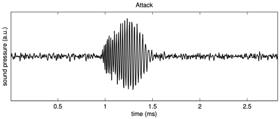
FIG. 2: The diffusion entropy $S(t)$ of the EEG increments of all the 19 channels for one of the 20 subjects considered in this study.

Credit : Dynamics of EEG Entropy : beyond signal plus noise

M. Ignaccolo, M. Latka, W. Jernajczyk, P. Grigolini and B.J. West (2009)

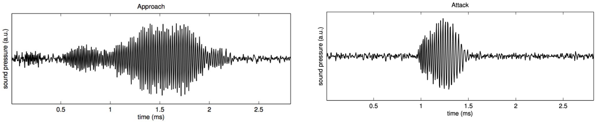
Back to bats





Ultrasounds emitted by a bat

Back to bats



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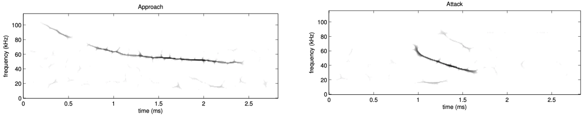
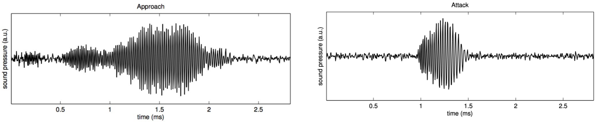


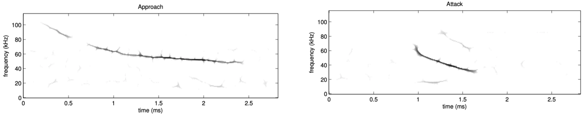
Time-Frequency analysis

Credit : *Explorations in Time-frequency Analysis* by P. Flandrin

Back to bats



Ultrasounds emitted by a bat



Time-Frequency analysis

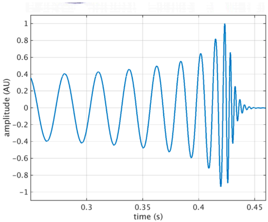
Credit : *Explorations in Time-frequency Analysis* by P. Flandrin

A more precise definition of Chirps :

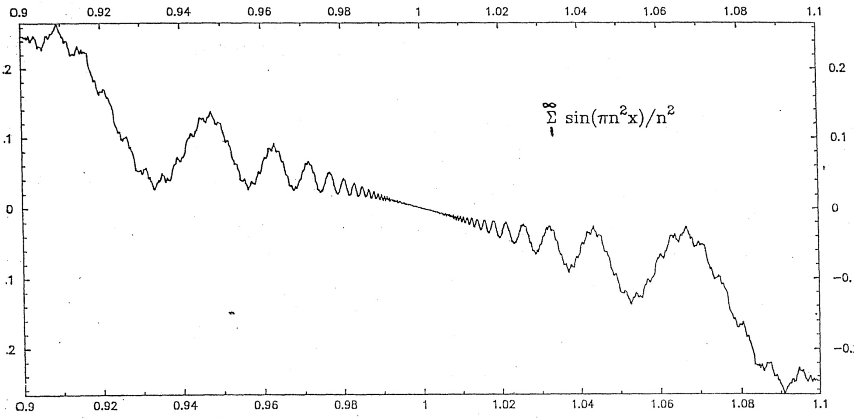
$f(t) = a(t)\cos(\varphi(t))$ where

$$\left|\frac{a'(t)}{a(t)}\right| << \varphi'(t) \text{ and } \frac{\varphi''(t)}{(\varphi'(t))^2} << 1$$

$$|t - t_0|^{-1/4}\cos(\omega|t - t_0|^{5/8} + \varphi)$$



Riemann's nondifferentiable function



Zoom of Riemann's function at a chirp

From G. Hardy to F. Broucke and J. Vindas (2023)

Riemann's nondifferentiable function and turbulence

A new and surprizing connexion between Riemann's function

$$\mathcal{R}(x) = \sum_{n=1}^{\infty} \frac{\sin(\pi n^2 x)}{n^2}$$

and turbulence was recently uncovered between Riemann's function and turbulence (V. Banica, A. Boritchev, D. Eceizabarrena, C. J. Garcia-Cervera, F. de la Hoz, R. L. Jerrard, D. Smets, L. Vega, and V. Vilaça Da Rocha) :

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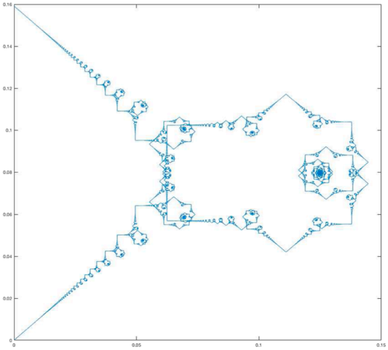
The complex-valued version

$$\phi(x) = i\pi x + \sum_{n=1}^{\infty} \frac{e^{i\pi n^2 x}}{n^2}$$

appears as the trajectory of the corners of polygonal vortex filaments that follow the binormal flow



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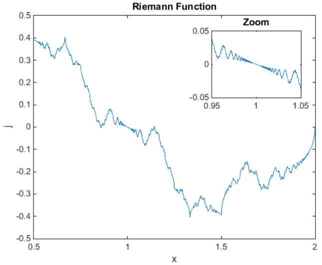

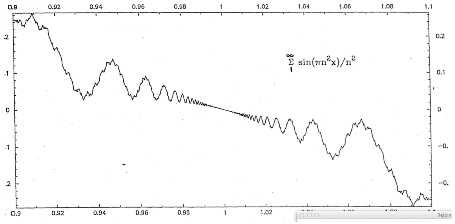

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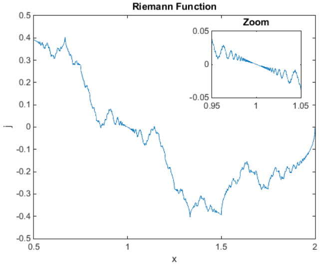

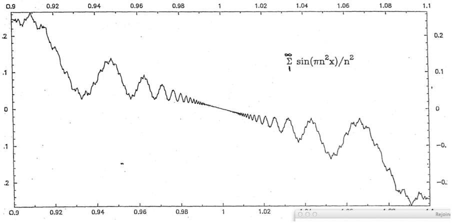

$$\mathcal{R}(\pi + x) = L(x) + x^{3/2} \left(\tilde{\mathcal{R}}\left(\frac{1}{x}\right) + x\tilde{\mathcal{R}}_1\left(\frac{1}{x}\right) + \cdots \right),$$

where (Y. Meyer and S. J.) :

- $L(x)$ is affine,
- $\tilde{\mathcal{R}}$ is (essentially) Riemann's function,
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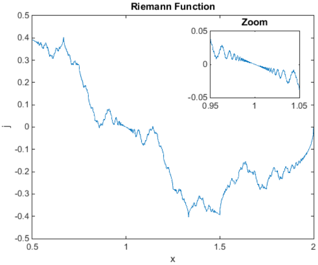

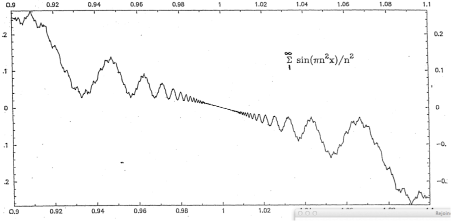

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How can we characterize such behaviors ?

Pointwise regularity exponent

$f \in C^{\alpha}(x_0)$ it there exist $C > 0$ and a polynomial P of degree $< \alpha$ such that

$$|f(x) - P(x - x_0)| \leq C|x - x_0|^{\alpha}$$

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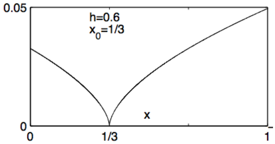
Cusps $\quad C_H(x) = |x - x_0|^H$



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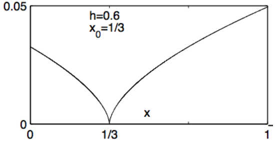


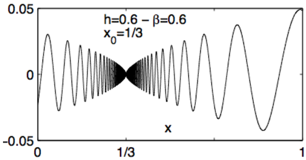
Cusps satisfy

$$h_f(x) = H \quad \text{and } h_{f(-1)}(x) = H + 1$$

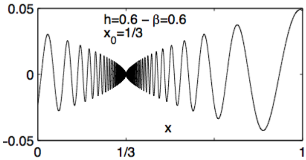
Pointwise regularity of oscillating singularities

Oscillating singularities

$$C_{H,\beta} = |x - x_0|^H \sin\left(\frac{1}{|x-x_0|^\beta}\right)$$



Pointwise regularity of oscillating singularities



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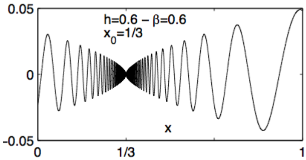

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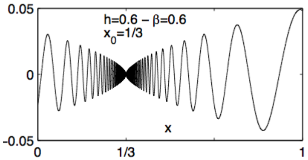
Pointwise regularity of oscillating singularities



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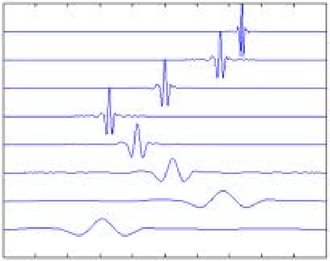
How can one take advantage of this difference ?

Use the wavelet characterization

of pointwise regularity

Orthonormal wavelet bases

A wavelet basis on $\mathbb{R}$ is generated by one smooth well localized oscillating wavelet ψ such that the $2^{j/2}\psi(2^j x - k), \quad j,k \in \mathbb{Z}$ form an orthonormal basis of $L^2(\mathbb{R})$

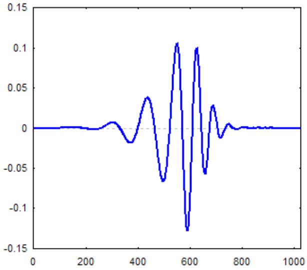


$\forall f \in L^2(\mathbb{R})$,

$$f(x) = \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} c_{j,k}\, 2^{j/2}\, \psi(2^j x - k)$$

with

$$c_{j,k} = 2^{j/2} \int f(x)\, \psi(2^j x - k)\, dx$$

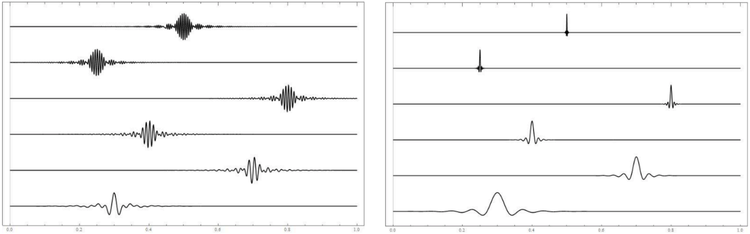


Daubechies wavelet

Example : $\psi = 1_{[0,1/2[} - 1_{[1/2,1[}$
(Haar wavelet)

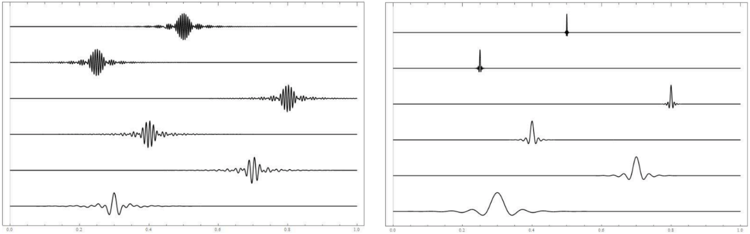
Credit : http ://www.kfs.oeaw.ac.at/content/blogcategory/0/502/lang,8859-1/

Orthonormal wavelet bases



Wilson vs. wavelet bases

Orthonormal wavelet bases



Wilson vs. wavelet bases

Some advantages of orthonormal wavelet bases :

- Characterization of “most” function spaces used in analysis (Hölder, Sobolev, Besov, ...)
- Characterization of pointwise regularity

Yves Meyer, Stéphane Mallat and Ingrid Daubechies

Wavelet leaders

Dyadic intervals : If $k \in \mathbb{Z}$ $\quad \lambda = \left[\frac{k}{2^j}, \frac{k+1}{2^j}\right)$

Wavelet coefficients : $c_\lambda = 2^j \int f(x)\psi(2^j x - k)dx$

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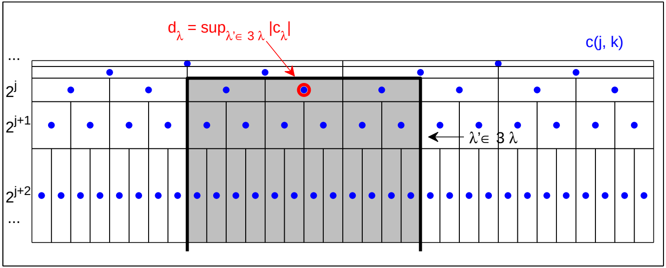
3λ is the interval of same center as λ and three times wider

The wavelet leaders of a function f are the quantities

$$d_\lambda = \sup_{\lambda' \subset 3\lambda} |c_{\lambda'}|$$



Characterization of pointwise smoothness

A function f is uniform Hölder if $\exists \varepsilon > 0 : f \in C^{\varepsilon}(\mathbb{R})$

or, equivalently, if $\exists \varepsilon > 0 : \quad \sup_{\lambda \in \Lambda_j} \leq C\, 2^{-\varepsilon j}$

Let $\lambda_j(x_0)$ denote the dyadic cube of width 2^{-j} that contains x_0

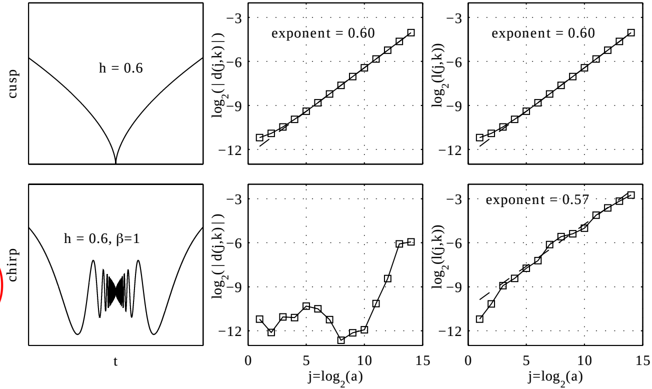
Theorem : (S. J.) If $f \in C^{\varepsilon}(\mathbb{R})$ for an $\varepsilon > 0$, then

$$\forall x_0 \in \mathbb{R}^d \qquad h_f(x_0) = \liminf_{j \to +\infty} \frac{\log(d_{\lambda_j(x_0)})}{\log(2^{-j})}.$$

Wavelet characterization of pointwise smoothness

$$|x - x_0|^h$$

$$|x - x_0|^h \sin\left(\frac{1}{|x - x_0|^\beta}\right)$$



Function

Regression on wavelet coefficients

Regression on wavelet leaders

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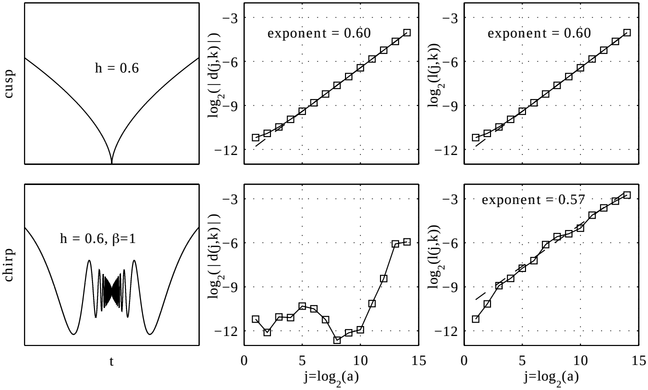

Function

Regression on wavelet coefficients

Regression on wavelet leaders

Log-log regressions of wavelet leaders yield a correct estimations of pointwise Hölder exponents for all kinds of singularities

Oscillating singularities

Mathematical examples

- Riemann series : Y. Meyer and S. J.
- Random wavelet series : P. Abry, R. Leonarduzzi, C. Melot, H. Wendt, S.J. and C. Esser, B. Vedel, S.J.
- Sample paths of some specific Lévy processes : P. Balança

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Multifractal analysis allows to derive information on the size of the sets of points x where

$$h_f(x) = H \quad \text{and } h_{f^{(-1)}}(x) = H + \beta + 1$$

Next challenges in applications

- Turbulence :
  with Patrice Abry (ENS Lyon)
  and Françoise Argoul
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- Build a common framework for the two approaches

If you want to know more on these topics

you are welcome to the next lectures

and the workshop !

(April 16-19)